Discrete Mathematics Unit

Grades 3 and 4

Red Lake County Central Elementary

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Executive Summary

Discrete Mathematics Unit Grades 3/4

This unit will cover the following standards and benchmarks.

***Algebra / Use single-operation input-output rules to represent patterns and relationships and to solve real world and mathematical problems. 3.2.1.1 Create, describe, and apply single operation input-output rules involving addition, subtraction, and multiplication to solve problems in various contexts**

***Algebra / Use single-operation input-output rules, tables, and charts to represent patterns and relationships and to solve real world and mathematical problems. 4.2.1.1 Create and use input-output rules involving addition, subtraction, and multiplication, and division to solve problems in various contexts. Record input or outputs in a chart or table.**

Starting at the beginning of the unit, students will use prior knowledge of multiplication and basic patterns to recognize more complex strategies to recognize patterns. Students will progress from basic patterns to recognizing rules for input output function machines as well as becoming fluent in writing equations to describe the rules of given patterns as well as for input output tables.

This unit was designed to help reinforce our school's MCA scores in Algebra.

Third Grade practice MCA questions

A table is shown

INPUT	Output
2	12
4	24
8	48

What is the output number when the input number is 12?

a. 2

b. 60

c. 72

d. 96

Gina buys a snack for $0.59. She pays with a $1 bill. She receives the fewest possible coins in change. What change does Gina receive?

a. 1 quarter, 1 dime, 1 nickel, and 1 penny

b. 2 quarters and 1 penny

c. 2 quarters, 1 nickel, and 4 pennies

d. 4 dimes and 1 penny

4th grade MCA's

A table is shown.

F	G
4	2
8	4
18	8

What rule was used to make the table?

A. **G=2f**
B. **G=f/2**
C. **G=f+2**
D. **G=2f=2**

Name_____

Pretest

1. **A table is shown**

INPUT	Output
2	12
4	24
8	48

What is the output number when the input number is 12?

a. 2

b. 60

c. 72

d. 96

2. Gina buys a snack for $0.59. She pays with a $1 bill. She receives the fewest possible coins in change. What change does Gina receive?

a. 1 quarter, 1 dime, 1 nickel, and 1 penny

b. 2 quarters and 1 penny

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d. 4 dimes and 1 penny

3. A table is shown.

F	G
4	2
8	4
18	8

What rule was used to make the table?

A. G=2f
B. G=f/2
C. G=f+2
D. G=2f=2

Day 1

Standard: Use single-operation input/output rules to represent patterns and relationships and to solve real- world and mathematical problems

3.3.1.2 Create, describe, and apply single operation input/output rules involving addition, subtraction, and multiplication to solve problems in various contexts

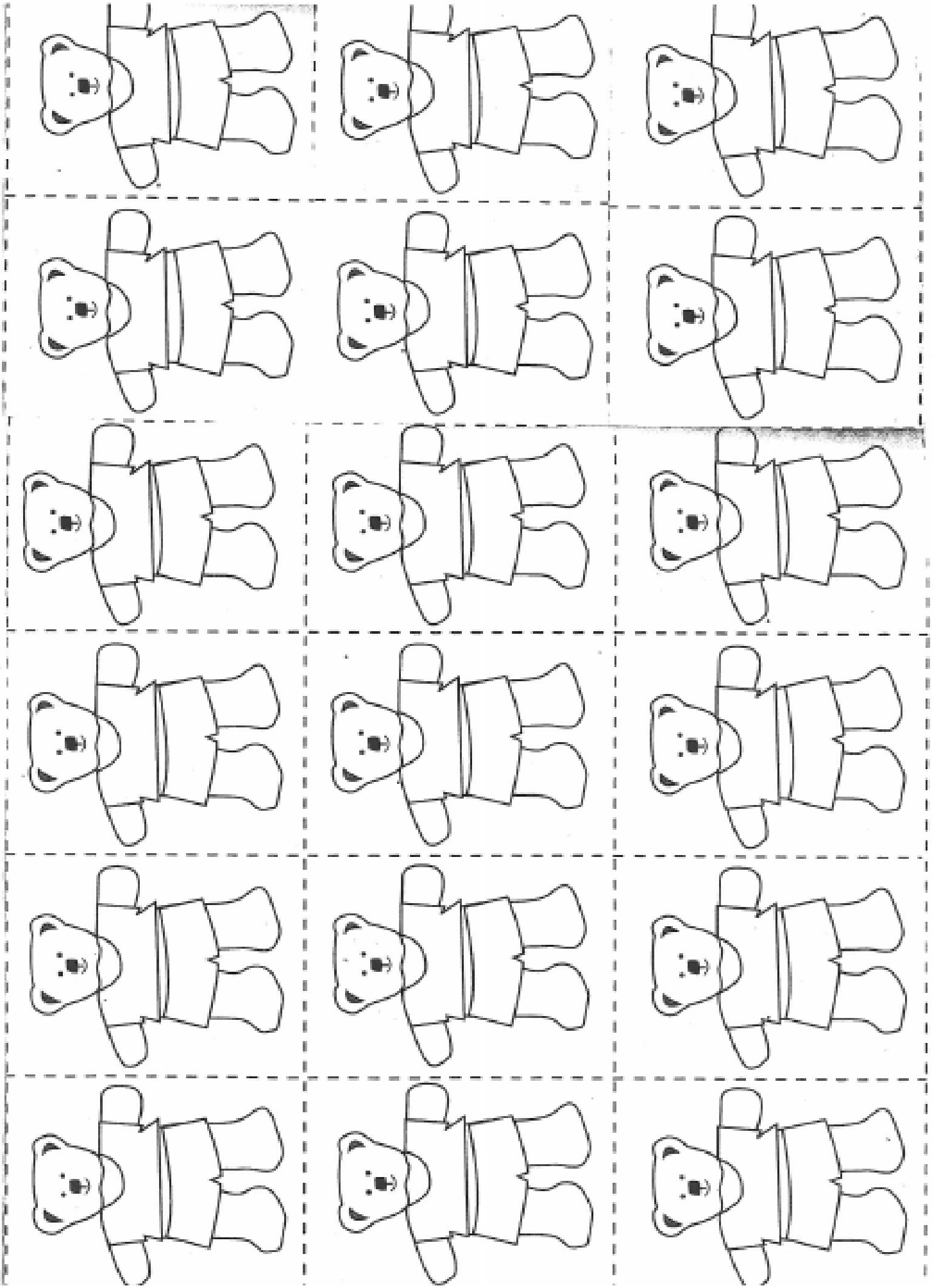
Launch: The students will take a pre-assessment. Once they finish their pre-assessment, the teacher will tell the students about a bear named Terry. Terry went shopping for some new clothes with his family. He bought one pair of black pants and one pair of brown pants. He bought three shirts, one orange shirt, one red shirt, and one blue shirt. Then Terry also bought 3 pairs of shoes, one yellow pair, one green pair, and one purple pair. The teacher will then ask the students to find how many possible different outfit combinations Terry can make to wear the first few weeks of school.

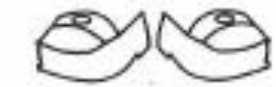
Explore: The teacher will pass out paper, crayons, glue, scissors and the copies of the outfits and shoes and bear patterns for the students to color and arrange in possible outfits for Terry. When they are done, they can arrange their outfits on paper to glue down.

Share:

The students will share their outfits as they have been completed with the class and explain how they came up with the different combinations of outfits and also how they organized their outfits to make sure that they did not have any repeating outfits.

Summarize: After the students have displayed their outfit combinations on their papers, the teacher will help the students to recognize the different possible ways to organize his outfits. This will show the students that there are multiple ways to display information. The teacher will ask the students how many different possible outfit combinations can be made with 2 pants, three shirts and 3 pairs of shoes. The students should answer 18.





Day 2

Standard: Algebra: Use single -operation input/output rules to represent patterns and relationships and to solve real world and mathematical problems

3.3.1.2 Create, describe and apply single- operation input/output rules involving addition, subtraction, and multiplication to solve problems in a various contexts

Objectives; The students will review their Terry Bear outfit combinations and use their prior knowledge of combinations and lists, to create a tree diagram.

The students will create a tree diagram, based on the possible number of outfit combinations.

The students will understand how a tree diagram is another way to represent their display of outfit combinations.

Procedure:

Launch: The students will come to class and all of their posters of Terry Bear's outfit combinations will be on display. The teacher will review with the class what was accomplished yesterday. The teacher will tell the students that today we are going to learn another way to represent the information that we made on our posters. Today we are going to create a tree graph to learn another way to show the outfit combinations that we created. The teacher will pass out paper for the students to use for their tree graphs. The teacher will create the graph on the board and the students will create their graph on their paper. How many shirts did Terry buy? How many pairs of pants? How many pairs of shoes? The teacher will draw the following tree graph on the board.

So how many outfit combinations is Terry Bear able to make? (18)

Explore:

The teacher would ask the students to draw several tree diagrams using a different number of shirts, pants, and pairs of shoes. The teacher would walk around the room, helping the students when needed and ask the students if they can recognize any patterns in their tree graphs. Hopefully, the students would recognize the pattern of multiplying the number of shirts x number of pants x number of pairs of shoes = the number of possible outfit combinations.

Share:

The teacher would call for a few volunteers to talk about their tree graphs and review their number of clothing and shoe choices with their number of total outfit combinations. The teacher would ask the students if they could use their calculators to figure out the possible number of outfit combinations. Student responses will vary and students may need help to recognize that the number of total outfit combinations is equal to the number of shirts x the number of pants x the number of pairs of shoes.

Summarize:

Near the end of class, the teacher will highlight the different tree diagrams that the students have created and ask them to explain how we can figure out the number of possible outfit combinations by using a calculator. Hopefully, the students will be able to verbally state that the total number of outfit combinations can be found by multiplying the number of shirts times the number of pants x the number of pairs of shoes.

Day 3

Standard:

Algebra: Use single operation input/output rules to represent patterns and relationships and to solve real-world and mathematical problems

3.3.1.2 Create, describe, and apply single-operation input/output rules involving addition, subtraction, and multiplication to solve real world problems in various contexts.

Objective: the students will represent patterns and identify patterns using unifix cubes, attribute blocks, cubes and the letters

Launch: The teacher will display many different patterns on the board and talk to the students about how patterns can be displayed in different ways. The teacher will explain that the students will work in groups to represent the various patterns at each table.

The students will work in groups at four different learning stations. While the students are working at the stations the teacher will be walking around the classroom and asking the students about their patterns. The teacher will be informally assessing the students and checking for their understanding of combinations and number patterns.

Explore:

At station 1, the students will work with unifix cubes to represent the patterns at that table. They will then record their patterns on paper.

At station 2, the students will create patterns using attribute blocks. They will then record their responses on paper.

At station 3, the students will work with geoboards and geobands to display the patterns at their tables. The student will again record their responses on paper.

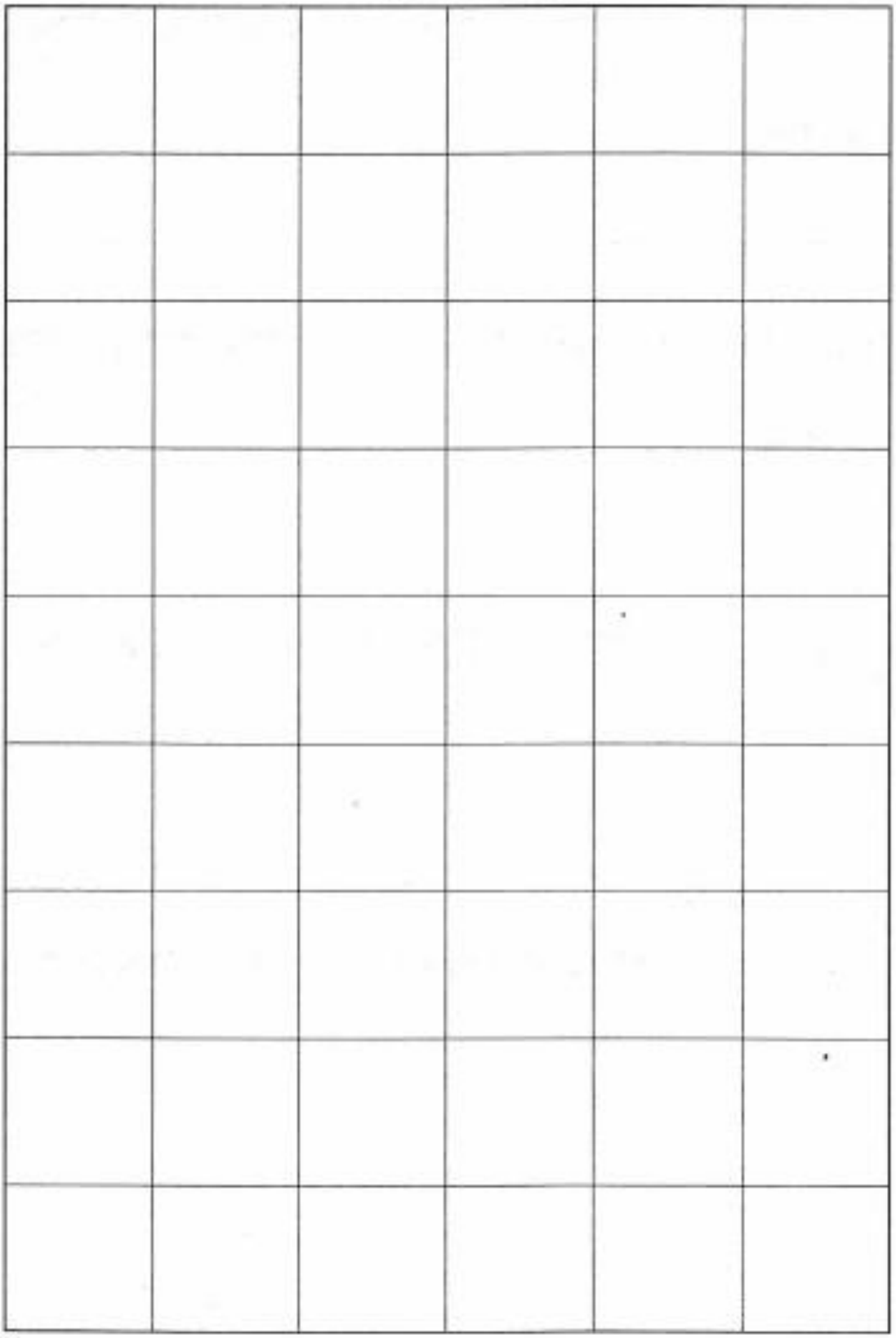
At station 4, the students will create their own letter patterns using the names of their classmates. They need to write the names of at least three classmates (they can choose their own name too) and write the names on the paper to arrange in a pattern. The students will then glue their name patterns on paper.

Share: The students will talk in their groups. We will put the name patterns on the board for everyone to see the many different combinations that were created with the names from our class.

Summarize: The teacher will ask the students which pattern was their favorite and which one was the most difficult to create. The teacher will then review how patterns can be represented in many different ways and that they be found anywhere in our environment.

One-Inch Grid Paper

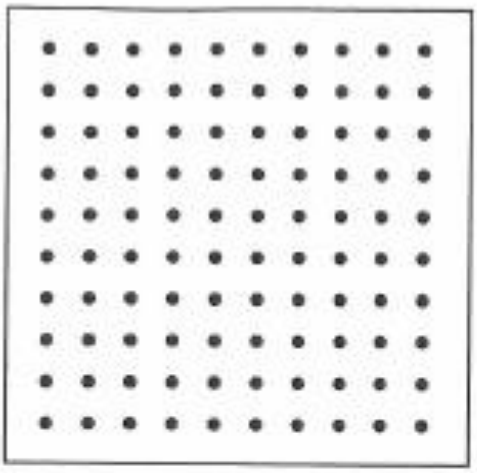
NAME DATE TIME

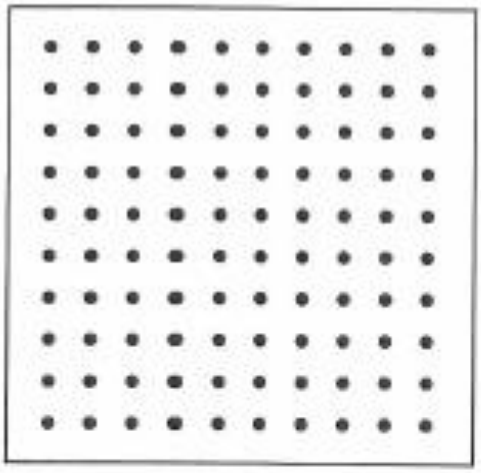


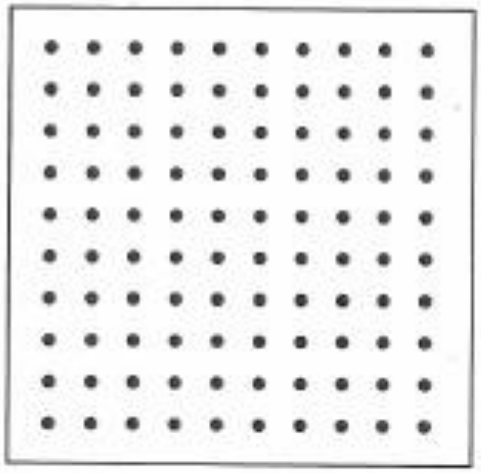
DM3

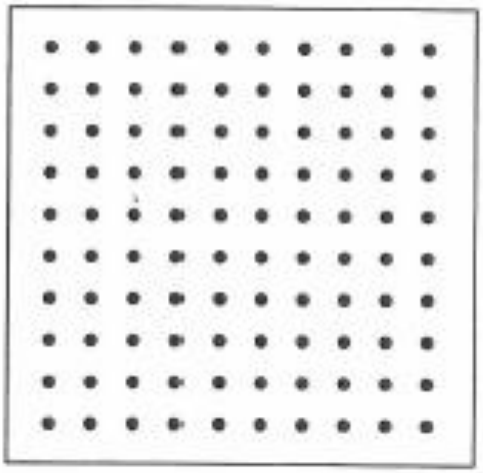
Array Dot Paper

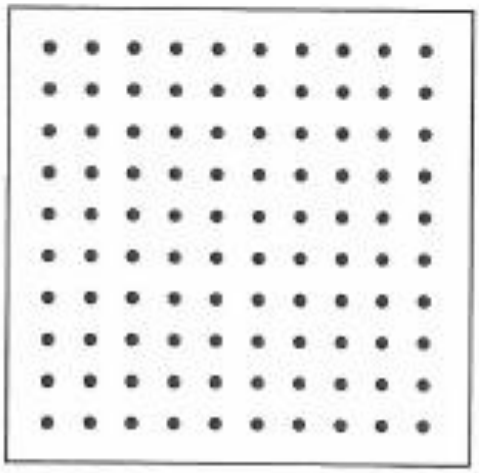
NAME DATE TIME

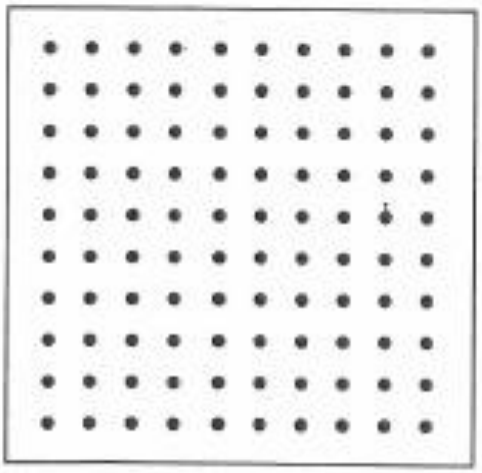












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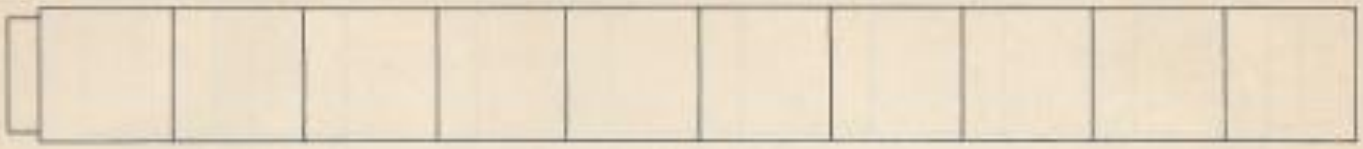

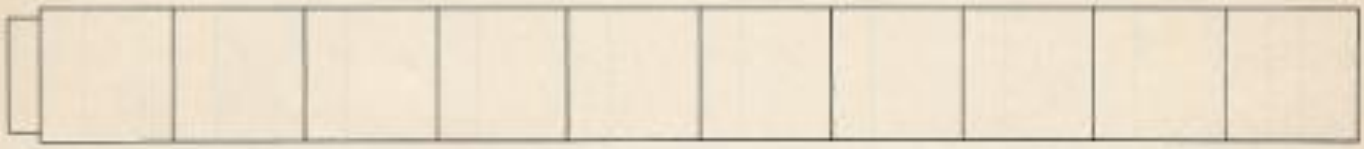

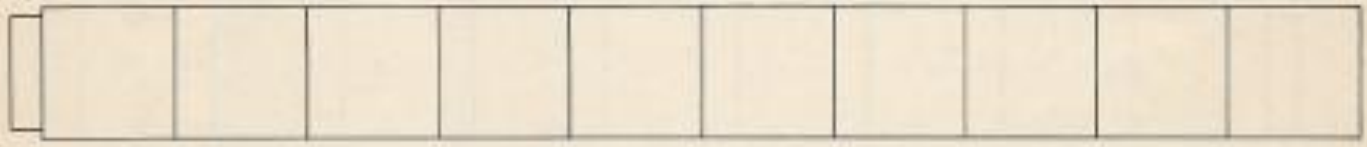

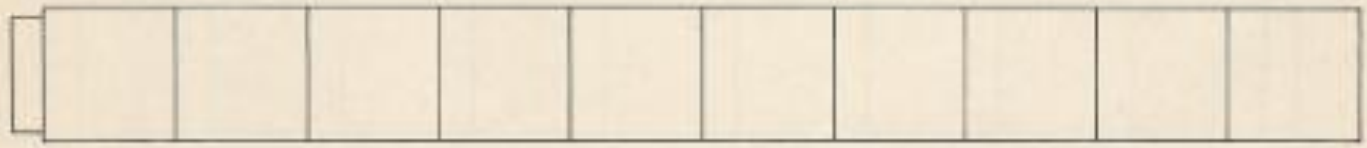

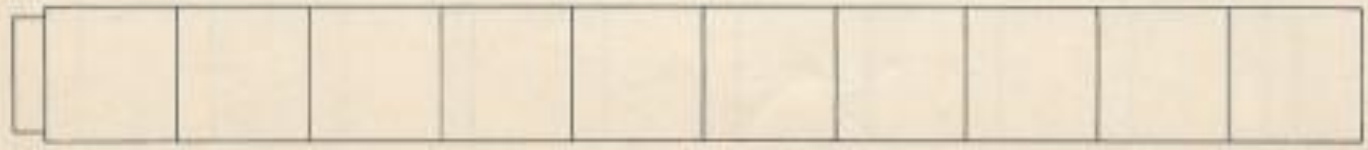

Day 4

Standard: Algebra: Use single –operation input/output rules to represent patterns and relationships and to solve real-world and mathematical problems.

3.3.1.2. Create, describe, and apply single-operation input/output rules involving addition, subtraction, and multiplication to solve problems in various contexts.

Objective: The students will solve number patterns and repeating patterns in a number sequence

Launch:

The teacher will write the numbers 1, 3, 5, 7 on the board and then ask the students what number would come next. The teacher will ask the students if they can recognize any patterns in the numbers and then ask them if they know the next number in sequence. Students may offer an idea that the pattern is skipping a number. The teacher will continue to write patterns on the board and ask the student to identify "What the rule is? for each pattern written.

Some of the patterns that could be used would include 1, 4, 7, 10, and 2, 6, 10, 14 and then the patterns 1, 3, 6, 10

Explore: The teacher would then give the students several repeating patterns in a number sequence (p.205) and ask them to identify what each pattern in the sequence that makes it a repeating pattern. The teacher would then ask the students to create 3 repeating pattern sequences of their own and have another student identify the repeating sequence.

Share: As the students have finished their problems they will discuss and compare their answers with the teacher. They will talk about the patterns that they observed and how students determined where the pattern in the number was and also what type of pattern it was, for example ABBBA or ABCD.

Summarize: The teacher will ask the students to review what kind of number patterns were talked about today and hopefully the students will be able to recall

the rule for each pattern was and also correctly label the patterns in sequence as ABCD, ABBBA, and ABBA.

Repeating Patterns in Number Sequences

Name _____

Picture after Picture–Activity 1

1. Think about the sequence below:

1	4	3	2	1	4	3	2	1	4	3	2	...

a. What pattern repeats to make this sequence?

b. What is the next term in the sequence?

2. Think about the sequence below:

1	4	1	1	4	1	1	4	1	1	4	1	...

a. What pattern repeats to make this sequence?

b. What is the next term in the sequence?

3. Think about the sequence below:

3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	...

a. What pattern repeats to make this sequence?

b. What is the next term in the sequence?

DM4

Day 5

Standard: Algebra: Use single-operation input/output rules to represent patterns and relationships and to solve real-world and mathematical problems

3.3.1.2 Create, describe, and apply single operation input/output rules involving addition, subtraction, and multiplication

Objective: the students will review number patterns and number sequences

Launch: The teacher will remind the students that a number sequence is just a list of numbers that repeats according to a set rule. Each number in the sequence is called a term and the first number is called the first term. The teacher would tell the students that if you start your sequence with the number one and the "rule" is that you are adding by four, the next number in the sequence would be five and the next number would be nine. The teacher will then put several number sequences on the board and have the students decide what the rule is and what number would come next in the sequence. Some possible sequences are 2, 5, 8, 11, and 14 or 5, 9, 13, 1 7, and 21.

Explore: The teacher will then give the each student a sheet of paper and have the students work with a partner to play "What's My Sequence? " The teacher will explain the game: Each student takes turns making up a number up for a starting point and then they will give the rule like add three. The other student must write down the next five terms in the sequence. The game will continue until both students have written down 5 sequences. The students can then check their number sequence patterns with a calculator to see if they are correct. The teacher will then pass out a Creating Number Sequences paper (p.217) to the students to have them work on individually.

Share: The students each share a number sequence that they created from their partner's directions. The students will be expected to share how each number in the sequence was created and answer any questions from their peers.

Summarize: The teacher will review the number sequences that were written on the board and ask the students to name the first term or fifth term in any given sequence. The students can identify which sequences were easier or harder to

solve. The teacher will also check for correct computations on the board and ask the students to hand in p. 217 to review for corrections.

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Creating Number Sequences

Name _____

Exploring Sequences Step by Step–Activity 1

1. Use the rule NOW + 3 = NEXT

Example:	
Rule:	NOW + 3 = NEXT
Place to begin:	Initial value = 2
Sequence:	2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, ...

a. What sequence do you get if you start with 1? List the first fifteen terms.

b. What sequence do you get if your initial value is 0? List the first fifteen terms.

c. What is the eighteenth term in the sequence in part (*b*)? _____

2. Look at the terms in each of the three sequences—the one in the example and the two that you generated in step 1.

a. Does one of the sequences include the number 26? _____
If so, which sequence? _____

b. Does either of the other sequences include the term 26? _____
If so, which sequence? _____

c. Does one of the sequences include the number 33? _____
If so, which sequence? _____

d. Does either of the other sequences include the term 33? _____
If so, which sequence? _____

e. Explain how you know that the number 57 will appear in one of the three sequences.

f. Which of the three sequences has 100 as a term? _____
Explain your answer.

g. Does every whole number appear in just one of these sequences? _____
How can you explain your answer or show that it is correct? Discuss this question with a partner before writing your ideas.

DM 5

Day 6

Standard:

Algebra: Use single-operation input-output rules to represent patterns and to solve real-world and mathematical problems.

3.2.1.1. Create, describe and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts.

Objectives:

- The students will sort, organize, and count small numbers of objects
- The students will use the addition principle of counting

Teacher preparation: For each group, copy the "Buttons for the Bucket" sheet onto four sheets of card stock in blue, red, yellow and green. Cut out the buttons and place them in a bucket or another suitable container. There should be 72 buttons for each group.

Launch:

The teacher will organize the students into groups of three and give each group a paper bag and a bucket containing all the buttons. To ensure that the groups of students have different numbers of buttons in the initial activities, the teacher will ask each group to designate one person to remove a handful of buttons from the bucket, put them in the paper bag, close it and set it aside until later. The teacher will ask each team to pour out the buttons that remain in its bucket, forming a pile that each team member can reach. The teacher will ask the students to imagine that they work for a sewing company that has just received a large shipment of buttons.

Explore:

The teacher will ask the students to imagine that they work for a sewing company that has just received a large shipment of buttons. The teacher will tell the

students that their first task is to determine the total number of buttons they have received. The teacher will ask each team to count the total amount of buttons in the bucket. The teacher will tell the teams to be creative in finding strategies to count their buttons.

Share:

One member of the team will explain how each team counted its buttons. For example, some groups may have divided the buttons into groups of ten, counted the number of tens and the number of extra buttons or some groups may have separated the buttons into smaller piles, with each team member counting one of the piles. The students should mention efficient strategies for counting.

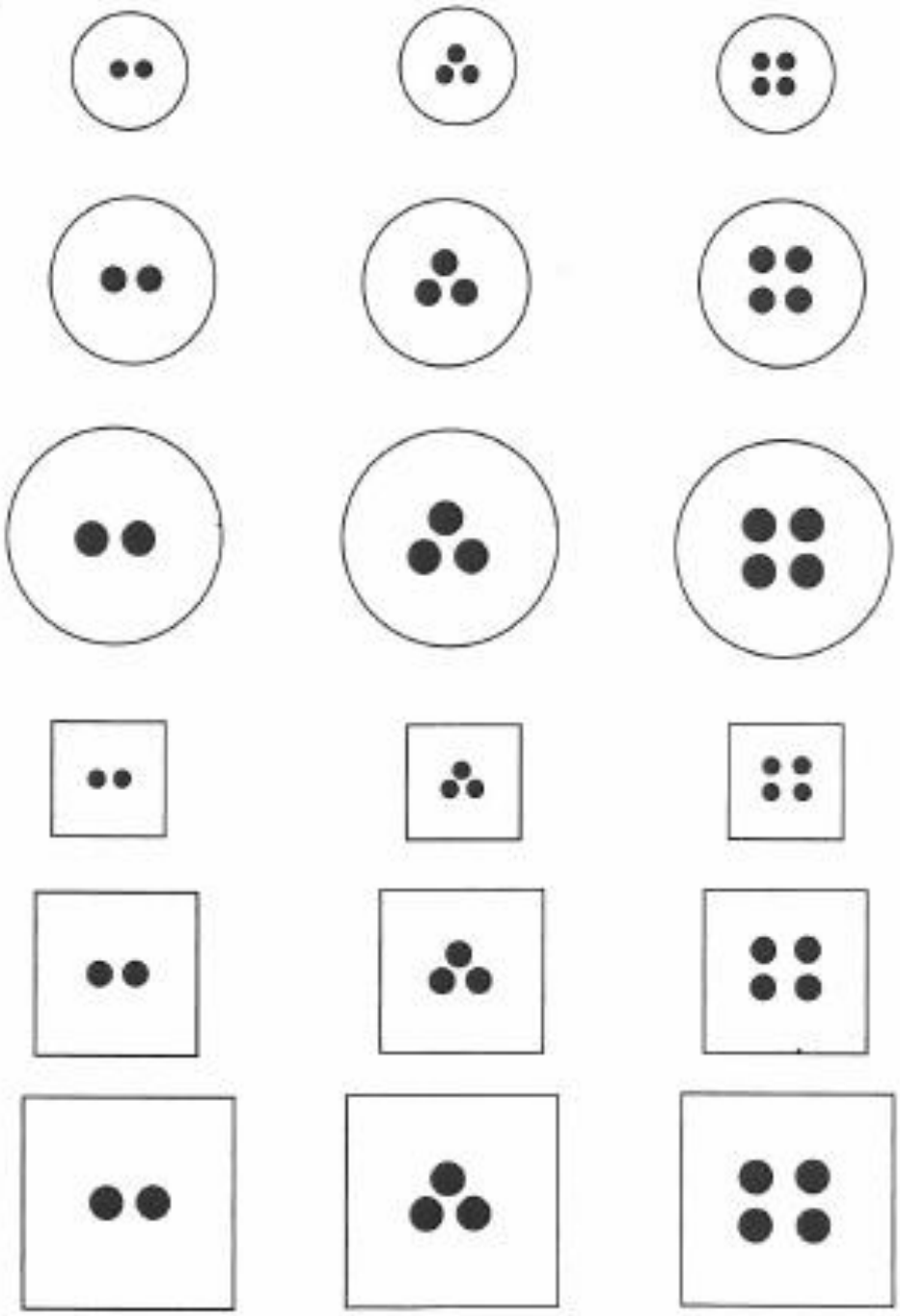
Summarize:

The teacher will tell the students that they sorted, organized and counted small objects. The teacher will tell students there are advantages and disadvantages to using some strategies when trying to count efficiently.

Buttons for the Bucket

Bucket of Buttons

To make the 72 buttons for each bucket, copy this template onto four sheets of poster board or card stock in different colors—blue, red, yellow, and green. Cut out the buttons, and place them in a bucket or another suitable container.



Day 7

Standard:

Algebra: Use single-operation input-output rules to represent patterns and to solve real-world and mathematical problems.

3.2.1.1. Create, describe and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts.

Objectives:

- The students will sort, organize, and count small numbers of objects
- The students will use the addition principle of counting

Launch:

The teacher will tell students to now imagine that the button manufacture mailed all the buttons in the same package instead of shipping them by button type, and this strategy has caused major problems for the sewing company. The teacher will tell the students that their task is to sort the buttons into different bins so that machine operators can quickly find the type of button that they need. The teacher will distribute six 36-inch lengths of string, each tied in a loop, to each group. The teacher will tell the students to arrange the strings in circles that do not overlap and imagine that each circle is a bin for buttons.

Explore and Share:

The students have more bins than they need, however, the teacher will avoid telling students in advance how many bins to use as this will limit their consideration of their sorting options.

The teacher will tell each group to (1) keep track of how its members sorted the buttons, (2) count the buttons in each bin, (3) and make a label for each bin. The teacher will give each group twelve strips of paper for the labels and tell the students that each label should explain the type of button that the bin contains. Each group will share their results. The teacher will ask each group to determine

the total number of buttons that were in their bucket. The students will explain how they chose to sort their buttons and the number of bins that their group used. Some teams may have sorted the buttons into two, three, perhaps, four or more bins. The teacher will ask the students to sort the bins into exactly three bins. If a group has already done this, ask the group to try to find a different way to sort the buttons into three bins. The students will share how they chose to sort their buttons. The teacher will point out that sorting by number of holes or by size involves exactly three different bins. The teacher will tell the students to put all of the buttons in the bins back into a single collection. The teacher will tell the students to put all the buttons with two-holes into a bin, label it "two-hole buttons", and count the number of buttons in the bin. The teacher will ask the students how many buttons are not two-hole buttons and how they determined the answer. The students may have solved this problem many different ways and the class will discuss each method used a well as these methods: subtracting the number of two-hole buttons from the total number of buttons that they determined on Day 6, counting the buttons that do not have two-holes, and counting the three and four-hole buttons and adding the sums. The students will put all of the buttons back into the bucket for a total of 72 buttons.

Summarize:

The teacher will tell the students that they can organize a large group of objects by dividing them into a smaller group according to different attributes.

Day 8

Standard:

Algebra: Use single-operation input-output rules to represent patterns and to solve real-world and mathematical problems.

3.2.1.1. Create, describe and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts.

Objectives:

- **The students will sort, organize, and count small numbers of objects**
- **The students will use the addition principle of counting**
- **The students will list all possibilities in counting situations**

Launch:

The teacher will give each student a piece of paper and a blue crayon, marker or colored pencil.

Explore and Share:

The teacher will ask the students to draw pictures, without looking at the buttons in the bucket, of all the small blue buttons that have two or three holes. Some students may not be able to draw all the possibilities. The teacher will call on individual students to describe what they drew. The responses should be: "One is round with two-holes, one is round with three-holes, one is square with two-holes and one is square with three-holes." The teacher will ask the students to draw pictures of all the blue buttons in the bucket that have two holes. The teacher will ask for the total. The students should find that the number of buttons is six. Three buttons are round-one large, one medium, one small; and three buttons are square-one is large, one medium, one small. The teacher will tell the students to draw all of the blue and round buttons. The teacher will ask the students to draw all of the possibilities that are round and have two-holes (12). The teacher will tell the students to check their answers by looking through the bucket to see whether they find any qualifying buttons in addition to those pictures they have drawn.

Summarize:

The teacher will tell the students that they learned how to sort and list a large group of objects into smaller groups.

Days 9 and 10

Standard:

Algebra: Use single-operation input-output rules to represent patterns and to solve real-world and mathematical problems.

3.2.1.1. Create, describe and apply single-operation input-output rules involving addition, subtraction and multiplication to solve problems in various contexts.

Objectives:

- **The students will sort, organize, and count small numbers of objects**
- **The students will use the addition principle of counting**
- **The students will list all possibilities in counting situations**
- **The students will organize, sort and count objects using Venn diagrams**

Launch:

The teacher will organize the students into groups of three. The teacher will pass out the bucket of buttons to each group.

Explore:

The teacher will ask each team to put aside all the yellow and green buttons. There should only be red and blue buttons (36) in the bucket. The teacher will tell the groups to count the blue buttons (18) and the big buttons (12). The teacher will give each team two string loops to place together so they overlap. The teacher will ask the students how many different regions the two circles form. The student's will most likely state that there are three regions. The teacher will point out that the space outside the circles is a region also. The teacher will tell the students that they are going to place each of the thirty-six buttons in one of the four regions. The teacher will give them strips of paper to use to label the circle at the left "blue buttons" and the right circle "big buttons". The teacher and students will refer to these circles as bins since the students will place the buttons inside them. The teacher will tell the students that the oval in the middle is part of both blue-button bin and the big-button bin, so the buttons that belong

inside the oval are those that are both blue and big. To model this, the teacher will take such a button and place it in this region in the bins of one team. The teacher will say that the crescent on the right is for buttons that are big but not blue. The teacher will place such a button there. The teacher will ask the students what kind of button they should place in the other crescent. The students should answer that the buttons in that crescent are blue but not big. The teacher will place an appropriate button there. The teacher will ask the students what kind of button goes outside the two bins. The students should answer that the region should have buttons that are not blue or big. The teacher will place an appropriate button there. Without looking, the teacher will pick one button at a time ask the students where it should be placed. The teacher will model this using three more buttons. The students will do this activity in their groups until all thirty-six buttons have been placed in one of four regions.

Share:

The students will count the number of buttons in each region and the teacher will record the answers on a chart:

	Left Crescent	Oval	Right Crescent	Outside
Blue and Red	12	6	6	12
Yellow				
Green				
Totals				

All the groups should agree that twelve (blue but not big) buttons are in the left crescent, six (blue and big) buttons are in the oval, six (big not blue) buttons are in the right crescent and twelve (neither blue nor big) buttons are in the outer region. The students should realize that the number of blue buttons (18) that they counted previously equals the number of big blue-buttons plus the number of not-big blue buttons. The teacher will ask the students how many students are either bog or blue but not big and blue. The students should say 6 + 12 = 18. The teacher will ask the students how many are big or blue or both big and blue. The students should say 12 + 6 + 6 = 24. The teacher will ask the students how many buttons are blue but not big (12) and how many buttons are not big and not blue (12). The

teacher will tell the students to take out the yellow buttons that they put aside. The teacher will tell the groups to count the yellow buttons (12) and place each of them in the appropriate region. The teacher will tell the students to count the number of yellow buttons in each region. The teacher will ask the students how many yellow buttons are in the left crescent (0), oval (0), right crescent (6), and the outside region (12). The teacher will record these answers on the chart:

	Left Crescent	Oval	Right Crescent	Outside
Blue and Red	12	6	6	12
Yellow	0	0	6	12
Green				
Totals				

The teacher will ask each group to imagine that all the groups have placed each green button in the appropriate region. The teacher will ask the groups how many buttons they would place in each of the four regions. The students should answer that they would find the same numbers for the green buttons that they found for the yellow buttons, since the number of the buttons of any one color is the same as the number of buttons of any other color. The teacher will record these answers on the chart:

	Left Crescent	Oval	Right Crescent	Outside
Blue and Red	12	6	6	12
Yellow	0	0	6	12
Green	0	0	6	12
Totals				

The teacher will ask each group to imagine that the teams have taken all the buttons in the original bucket of buttons and placed each button in the appropriate region. The teacher will ask the students how many buttons would be in each of the four regions. This should enable each group to realize that the number of buttons in the outside region, for example, is the number of blue and red buttons in the outside region, plus the number of yellow buttons in the outside region, plus

the (imagined) green buttons in the outside region, for a total of thirty-six buttons. The teacher will record the answers on the chart:

	Left Crescent	Oval	Right Crescent	Outside
Blue and Red	12	6	6	12
Yellow	0	0	6	12
Green	0	0	6	12
Totals	12	6	18	36

The teacher will tell the groups to return all the buttons to the bucket, and then ask each student to take a handful of buttons (about 20) from the bucket. The teacher will handout the "Handful of Buttons" activity sheet and ask the students to repeat the preceding activity individually. After everyone has finished the assignment, each student will select a partner to check his/her work.

Summarize:

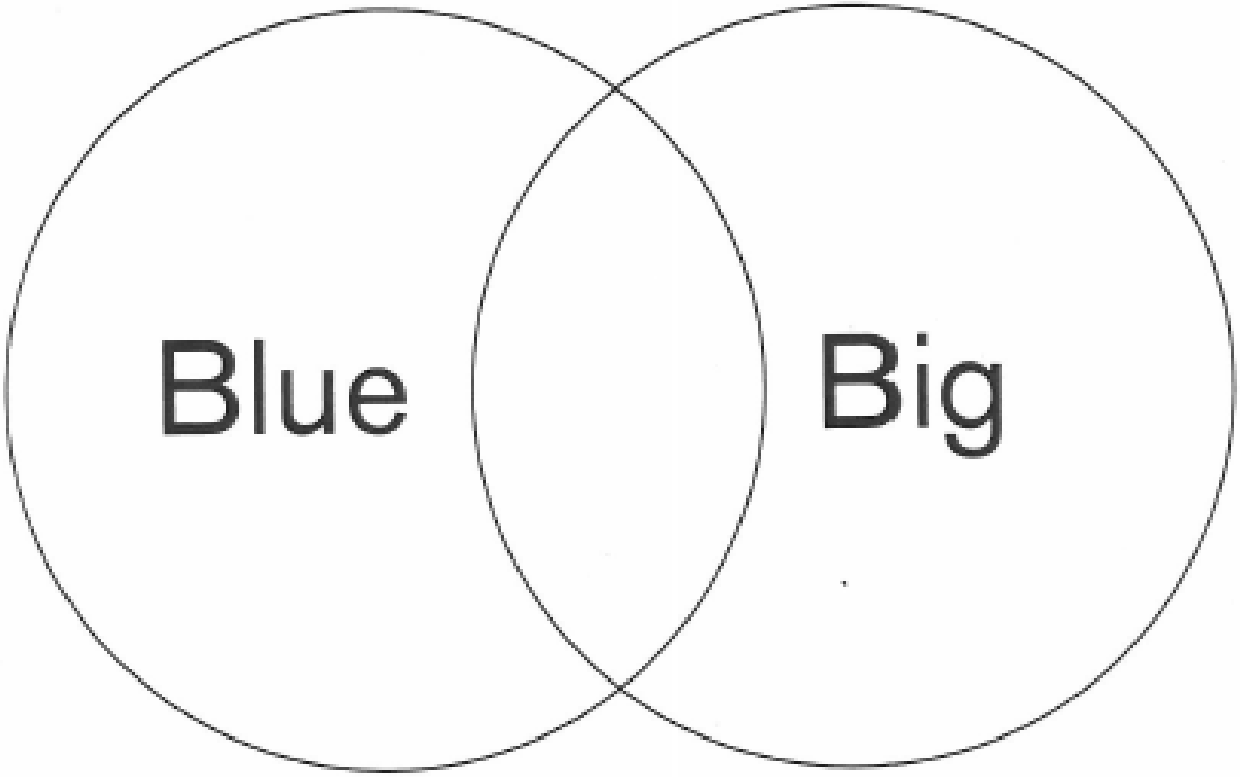
The teacher will tell the students that they learned how to organize, sort and count objects using a Venn diagram.

Handful of Buttons

Name _____

Bucket of Buttons–Activity 4

Take a handful of buttons from the bucket. Place each button in the correct space in the Venn diagram.

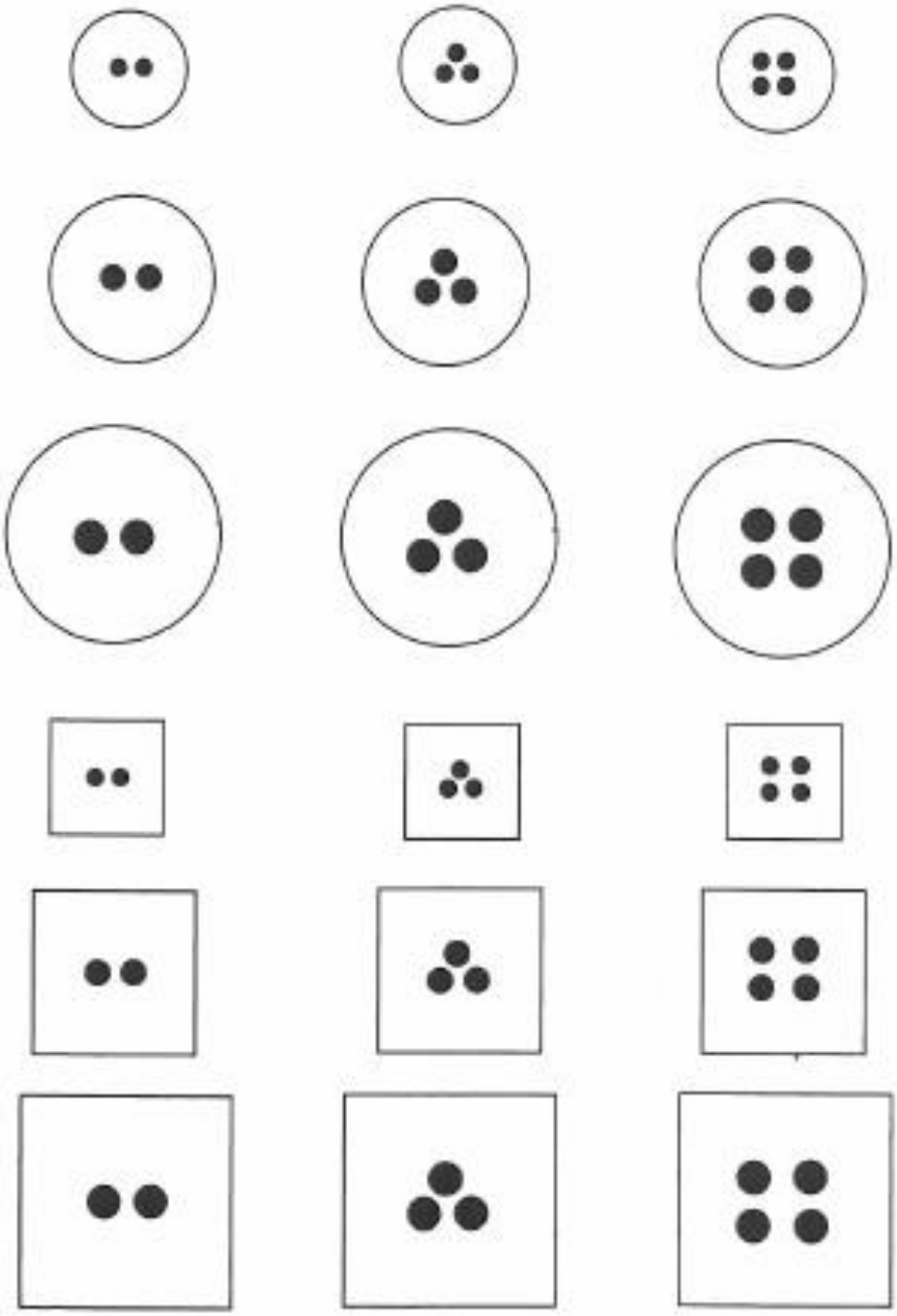


1. How many buttons in your handful are blue? _____
2. How many buttons in your handful are big? _____
3. How many buttons in your handful are blue and big? _____
4. How many buttons in your handful are blue but not big? _____
5. How many buttons in your handful are big but not blue? _____
6. How many buttons in your handful are not blue and not big? _____
7. How many buttons in your handful are blue or big? _____
8. How many buttons in your handful are big or blue, but not both? _____

Buttons for the Bucket

Bucket of Buttons

To make the 72 buttons for each bucket, copy this template onto four sheets of poster board or card stock in different colors—blue, red, yellow, and green. Cut out the buttons, and place them in a bucket or another suitable container.



Day 11 Discrete Mathematics

MN Standard: Algebra / Use single-operation input-output rules, tables, and charts to represent patterns and relationships and to solve real world and mathematical problems. 4.2.1.1 Create and use input-output rules involving addition, subtraction, and multiplication, and division to solve problems in various contexts. Record input or outputs in a chart or table.

Learning Objectives:

- Students will look for mathematical structures such as categories, patterns, and properties
- Students will use structures to solve problems and answer questions
- Students will create and justify rules, shortcuts, and generalizations

Launch: The teacher will display the following table on the document camera and tell students that the average person in the United States uses about 50 gallons of water a day. The teacher will tell students to complete the table below using that information.

In (days)	Out (gallons)
2	100
6	
10	
30	
365	

When students have completed, have students compare their completed tables. The rule is multiply by 50.

The teacher will remind students how a function machine works. 1) A number (the input) is dropped into the machine. 2) The machine changes the number according to a rule. 3) A new number (the output) comes out the other end.

Explore: The teacher will give students a blank "What's my rule tables?" The teacher will demonstrate each type of "What's my Rule?" while students work in groups or pairs to solve the tables.

Type 1 Rule: Subtract 15

In	Out
30	
90	
65	
110	

Type 1, the rule and sample inputs are known and the outputs must be determined.

Type 2: Add 100

In	Out
	350
	120
	665
	421

Type 2, the rule and sample outputs are known and the inputs must be determined.

Rule: ?

In	Out
7	49
2	14
9	63

Type 3, the sample inputs are known and the rule must be determined.

Type 4 Rule: ?

Pounds	Cost
1	$2.00
2	
5	$10.00
11	$

Type 4, Sample inputs and outputs are known and the missing numbers and rule must be determined.

Share: After students have worked to complete, "What's my Rule?" tables, invite volunteers to share their answers. Focus discussion on the patterns students found in the tables that are not stated in the rules.

Summarize: The teacher will review how a function machine works, **restating,** 1) A number (the input) is dropped into the machine. 2) The machine changes the number according to a rule. 3) A new number (the output) comes out the other end. The teacher will also review the four types of function tables and what strategies the students used to solve them. Type 1, the rule and sample inputs are known and the outputs must be determined. Type 2, the rule and sample outputs are known and the inputs must be determined. Type 3, the sample inputs are known and the rule must be determined. Type 4, Sample inputs and outputs are known and the missing numbers and rule must be determined.

In (days)	Out (gallons)
2	100
6	
10	
30	
365	

What's My Rule?

What's My Rule?

Day 12 Discrete Mathematics

Lesson Objectives: Students begin by looking at three groups of problems and finding what each group of problems has in common. Students then play a game with a function machine in which they will have to figure out unknown rules for different function tables. For independent practice, students match function tables to their rules and complete function tables with missing outputs. Advanced learners create input-output tables to answer real-world problems. Struggling learners use modified input-output tables to find rules and missing output.

MN Standard: Algebra / Use single-operation input-output rules, tables, and charts to represent patterns and relationships and to solve real world and mathematical problems. **4.2.1.1 Create and use input-output rules involving addition, subtraction, and multiplication, and division to solve problems in various contexts. Record input or outputs in a chart or table.**

The students will know…

- How to read a function table.
- How to find the rules for function tables.
- How to find missing input or output for function tables.

The students will be able to…

- Find rules for function tables.
- Find missing input or output for function tables.

Launch: Write the following groups of problems on the board.

2 x 10 = 20	50 – 17 = 33	15 + 9 = 24
9 x 2 = 18	21 – 17 = 4	9 + 18 = 27
4 x 2 = 8	18 – 17 = 1	2 + 9 = 11
2 x 25 = 50	29 – 17 = 12	9 + 30 = 39

Ask students to look at each group of problems and find what each group has in common. Have students write their answers on scratch paper. If necessary, allow students to discuss the groups of problems with other students around them. When all students have decided on their answers, ask students to share what they think the first group of problems has in common. Make sure they understand that all of the problems are multiplication problems and include 2 as one of the factors. Repeat this process for the remaining two groups of problems. Students should see that the problems in the second group are subtraction problems that have 17 as the subtrahend, and all of the problems in the third group are addition problems that have 9 as one of the addends.

Explore: ** Prior to the lesson, use an empty box of tissues or shoe box to create a function machine. Wrap the outside of the box in butcher paper. Cut a large slit in both the top and bottom. Write "Input" on the top of the box above the slit. Write "Output" on the bottom of the box above the slit. Write "Function Machine" on the side of the box that will face students. In addition to creating the function machine, write the following function rules on separate index cards: ÷ 2, x 4, x 30, + 11, - 5, and x 6.**

1) Show students the function machine that you created prior to the lesson. Explain to students how the function machine works. A number goes in as input in the top of the machine. Once inside the machine, a rule known only by the function machine changes the number. Another number is produced as output that comes

out of the bottom of the machine. Tell students that their job is to figure out the rule that the function machine is using, knowing only the input and output numbers. Draw an input-output table on the board like the one shown below. Tell students that the input-output table is used to show the input and output of the function machine and to help them figure out the function machine's rule.

Input	Output

2) Tell students that you know the function machine's rule (input + 20 = output), and they will have to figure out what the rule is. Choose a student and ask him/her to give you some input (a number). Record the number in the first row of the input column of the table on the board. Add the input to 20 without telling students what you did, tell the students what the output is, and record it in the first row of the output column. Choose another student to provide a number for input. Emphasize that the second student's number should be a bit larger than the first student's input number because the numbers in the input column should be listed from least to greatest. Add 20 to the second students' number, and record it in the second row of the output column. Repeat this process until the entire table is complete. Ask students to look at each row to try and figure out what the rule of the function table is. Elicit responses from students, making sure that they understand that the number in the input column was added to 20 to produce the number in the output column. Write the rule, input + 20 = output, on the board.

3) Draw another input-output table on the board. Call a student up to the front of the room to hold the function machine, and explain that student will know the rule for function machine and the rest of the class will have to figure out the rule. Show the student one of the rules that you wrote on one of the index cards. Ask one of the other students in the class to provide a number as input, and record it on the table on the board. If necessary, give the student providing input some parameters. For example, if the rule is ÷ 2, tell students that all of the numbers for input must be even. Have the student holding the function machine say what the output is, and record it on the table. Choose other students to provide input until the table is complete. Have students look at the input-output table and figure out what the rule is. Elicit responses until a student mentions that the number in input is divided by 2 to produce the number in output. Record the rule as input ÷ 2 = output.

4) Draw another function table on the board, and choose another student to hold the function machine. Show them another index card with a rule on it. Repeat the process of calling on students in the class to provide the input, recording it on the table with the output given by the student holding the function machine. Repeat this process for all of the rules written on the index cards.

Share: Ask students: "What were the rules that were easier to decipher than others?" "Why were they easier?" Also what made others more difficult and why? Have students come up with other functions that they could add to the table. Discuss with students the strategies to solve an input output table. Also ask for suggestions on where in real life we see input output tables and why we need these in our lives. Give each student a copy of the Independent Practice, explain the directions, and allow students to work independently.

Summarize: The teacher will review how a function machine works, restating, 1) A number (the input) is dropped into the machine. 2) The machine changes the number according to a rule. 3) A new number (the output) comes out the other end. The teacher and students will also discuss the different strategies used to solve

the function tables. The teacher will also state how students are looking for what each set of numbers has in common.

Name: _____

Model Functions

Directions: Read each problem below. Create an input-output table that answers the question. Identify the rule for the input-output table.

1. An amusement park is building a new roller coaster. Each car for the roller coaster has 6 wheels. How many wheels will they need to order for 2 cars? 4 cars? 8 cars? 12 cars?

Input	Output

Rule for the table: _____

2. Each time Juan earns money for doing chores; he puts $2 into savings and keeps the rest to spend. How much money will have to spend if he earns $7? $10? $15? $20?

Input	Output

Rule for the table: _____

3. Tyson and Rebecca babysit to earn money. They split the money they make in half. How much money will one girl make if the girls earn $20? $30? $36? $44?

Input	**Output**

Rule for the table: _____

4. One package of party invitations contains 12 invitations. How many invitations are in 7 packages? 9 packages? 12 packages? 14 packages?

Input	**Output**

Rule for the table: _____

Day 13 Discrete Mathematics

MN Standard: Algebra / Use single-operation input-output rules, tables, and charts to represent patterns and relationships and to solve real world and mathematical problems. 4.2.1.1 Create and use input-output rules involving addition, subtraction, and multiplication, and division to solve problems in various contexts. Record input or outputs in a chart or table.

Lesson Objectives:

The students will know…

- How to read a function table.
- How to find the rules for function tables.
- How to find missing input or output for function tables.

The students will be able to…

- Find rules for function tables.
- Find missing input or output for function tables.

Launch: The teacher will tell the students that today we will be rotating between three different stations today, that will focus on reviewing the input output rules that we have been learning. Remind students how a function machine works, 1) A number (the input) is dropped into the machine. 2) The machine changes the number according to a rule. 3) A new number (the output) comes out the other end. The teacher will open discussion on the different strategies used to solve the function tables this week.

Explore:

on the left and fold along the orange and yellow lines. Each flap has a function. Under each flap, students will make a table using the fold. They will write X on the left side and y on the right side and fill in values for X and solve for Y.

Station 2 / Match-Up - Directions: Students will cut out the tables and functions. Students will work independently, in pairs, or groups to match the tables to the function. ** Not all functions will be used**

Station 3 / Independent Practice - Students will work to complete the tables according to the function they are given.

Share: After students have rotated throughout the stations. Students will be invited to share some of the functions they created in the Function Foldable with the class, students will also share which functions were more difficult to "match up." Students will also ask for clarification on any aspect that caused any confusion.

Summarize: The teacher will ask "What made your tasks easy?" "What made them difficult?" The teacher will also discuss the importance of recognizing a pattern when solving function machines.

X	Y
3	12
4	16
7	28
9	36

X	Y
35	29
28	22
16	10
9	3

X	Y
7	35
9	45
12	60
15	75

X	Y
24	8
18	6
15	5
9	3

$y = 4x$

$y = x - 6$

$y = 5x$

$y = 4 + x$

$y = x \div 3$

$y = x + 28$

$y = x \div 6$

$y = 5x \div 2$

$y = 4x - 2$

$y = x^2$

Throw This Section Away

X	Y
2	
3	
4	
5	

$y = x + 3$

X	Y
3	
4	
6	
7	

$y = 5x$

X	Y
3	
4	
5	
6	

$y = x - 2$

X	Y
2	
3	
8	
9	

$y = 4x$

X	Y
2	
3	
4	
5	

$y = 2x + 5$

X	Y
3	
6	
9	
12	

$y = x \div 3 + 2$

X	Y
3	
4	
5	
6	

$y = 4x - 7$

X	Y
2	
3	
8	
9	

$y = 8x - 10$

X	Y
4	11
5	13
8	19
11	25

X	Y
46	30
38	26
24	19
14	14

X	Y
2	8
3	14
6	32
7	38

X	Y
4	8
5	11
6	14
9	23

$y = 2x + 3$

$y = 6x - 4$

$y = x + 2$

$y = x + 7$

$y = x \div 2 + 7$

$y = 3x - 4$

Day 14

MN Standard: Algebra / Use single-operation input-output rules, tables, and charts to represent patterns and relationships and to solve real world and mathematical problems. 4.2.1.1 Create and use input-output rules involving addition, subtraction, and multiplication, and division to solve problems in various contexts. Record input or outputs in a chart or table.

Lesson Objectives:

Students will:

- understand the concepts of input and output.
- formulate a rule for their answer.
- test their rule sufficiently to be sure that it works.
- work with applying an equation to letters of the alphabet to code words and messages.

Launch: The teacher will explain that today students will be using the alphabet as our input and applying a rule to get a numerical output. The process will be similar to previous days. The following will need to be on the board or under a document camera so that the students can easily refer to it:

Original Letter to Number Equivalencies:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

Start with a simple word, cat. The input would be 2, 0, 19. Devise your rule and apply it to your input. Cat has now changed to something else. Do a few more easy words with students to ensure that they understand the process.

Explore: Then have them to do their name, sports teams, games, family members, etc…using different rules to get the output. They will decide on their own rule. They can then exchange papers with someone to see if they can figure out each other's rule.

Share: Students will share their creations. The discussion will focus on the patterns students created in the tables and the other students will share how they figured out the missing rule or rules.

Summarize: The teacher will ask students, "How they know their rule works?" "What do we need to do to check to see if the rule "fits" the table?"

Day 15 Discrete Math

MN Standard: Algebra / Use single-operation input-output rules, tables, and charts to represent patterns and relationships and to solve real world and mathematical problems. 4.2.1.1 Create and use input-output rules involving addition, subtraction, and multiplication, and division to solve problems in various contexts. Record input or outputs in a chart or table.

Learning Objectives:

- Students will look for mathematical structures such as categories, patterns, and properties
- Students will use structures to solve problems and answer questions
- Students will create and justify rules, shortcuts, and generalizations

Launch: The teacher will tell students that today they will be using pattern blocks to determine the connection between the number of squares and triangle and the number of sides they have. Students will also do an activity based on the book, ***Two of Everything*** by Lily Toy Hong.

Explore: Hand out page 89 and pattern blocks. Students will work together to fill in the table to figure out the number of sides for different numbers of squares and different numbers of triangles, while writing an equation that fits the rule. When students finish, partners will read the book, ***Two of Everything*** by Lily Toy Hong. Partners will imagine that they drop five coins in a pot and then drop that

new number in the pot again the next day. Partners will estimate how many coins they will take out of the pot on day 10, and write that estimate on a piece of paper. Partners will make a "What's my Rule?" table on paper and calculate the number of coins they will take out of the pot on day 10.

Share: Students will share if the pattern blocks were helpful in determining the number of sides for the different amount of shapes as well as discuss how close their estimate to the real number of coins was on day 10. Also was there anything that surprised you about how the number of sides or coins changed from day to day? We can also estimate and calculate to find what day the pot will produce at least 1,000,000 coins.

Summarize: The teacher will reinforce that using diagrams, tables, charts, and other tools will help make your answers more concrete and definite. Also discuss once again the strategies of setting up equations and what information is available and what is missing to help determine patterns and rules.

"What's My Rule?" Number of Sides

Lesson 2-13

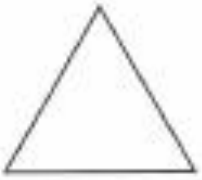
NAME DATE TIME

(1) Use square pattern blocks to help you complete the table.

Number of Squares	Number of Sides
1	4
2	8
3	
5	
7	
8	

(2) Suppose there are 12 squares. Explain how to find the number of sides without counting.

(3) Use triangle pattern blocks to help you complete the table.



Number of Triangles	Number of Sides
1	3
2	6
	15
	12
	9
	18

(4) Suppose there are 30 sides. Explain how to find the number of triangles without counting.

Name_____

Posttest

1. **A table is shown**

INPUT	Output
6	24
8	32
9	36

What is the output number when the input number is 11?

a. 22

b. 33

c. 44

d. 55

2. Bill buys a snack for $0.69. She pays with a $1 bill. She receives the fewest possible coins in change. What change does Bill receive?

a. 1 quarter, 1 nickel, and 1 penny

b. 1 quarter and 1 penny

c. 2 quarters, 1 nickel, and 4 pennies

d. 3 dimes and 1 nickle

3. A table is shown.

F	G
16	4
32	8
40	10

What rule was used to make the table?

A. G=4f
B. G=f/4
C. G=f+4
D. G=2f=4

Source Citations

Days 6-10 of Discrete Mathematics Unit Plan:
Valerie A. DeBellis, Eric W. Hart, Margaret J. Kenney, and Joseph G. Rosenstein. Navigating through Discrete Mathematics in Pre-kindergarten-Grade 5. The National Council of Teachers of Mathematics Inc., 2009, pp. 164,167,19-29.

Days 1, 2, 3,4
DeBellis, Valerie A., Eric W. Hart, Margaret J. Kenney, and Joseph G. Rosenstein. Navigating Through Discrete Mathematics in Pre-kindergarten-Grade 5 The National Council of Teachers of Mathematics Inc. 2009 p 35-45, 205, 217

Days 11 & 15 Everyday Mathematics, McGraw Hill
Day 12 Study Island, 2015, Teacher Lesson Plan Input Output function